Chapter - Sets

Topic-1: Sets, Types of Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1 MCQs with One Correct Answer

1. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then **[2011]**

   (a) $P \subset Q$ and $Q - P \neq \phi$ (b) $Q \not\subset P$
   (c) $P \not\subset Q$ (d) $P = Q$

2. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to **[2010]**

   (a) 25 (b) 34 (c) 42 (d) 41

Topic-2: Venn Diagrams, De Morgan's law, Practical Problem

1 MCQs with One Correct Answer

1. If X and Y are two sets, then $X \cap (X \cup Y)^c$ equals. **[1979]**

   (a) X (b) Y (c) ϕ (d) None

6 MCQs with One or More than One Correct Answer

2. In a college of 300 students every student reads 5 newspapers and every newspaper is read by 60 students. The number of newpapers is **[1998 - 2 Marks]**

   (a) at least 30 (b) at most 20
   (c) exactly 25 (d) none of these

10 Subjective Problems

3. Suppose $A_1, A_2, \ldots\ldots A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots\ldots B_n$ are n sets each with three elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$. Assume that each element of S belongs to exactly ten of the A_i' s and to exactly nine of the B_j' s. Find n. **[1981 - 2 Marks]**

4. (i) Set A has 3 elements, and set B has 6 elements. What can be the minimum number of elements in the set $A \cup B$? **[1980]**

   (ii) P, Q, R are subsets of a set A. Is the following equality true ?
   $R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$?

   (iii) For any two subset X and Y of a set A define
   $X \circ Y = (X^c \cap Y) \cup (X \cap Y^c)$
   Then for any three subsets X, Y and Z of the set A, is the following equality true.
   $(X \circ Y) \circ Z = X \circ (Y \circ Z)$?

5. An investigator interviewed 100 students to determine their preferences for the three drinks : milk (M), coffee (C) and tea (T). He reported the following : 10 students had all the three drinks M, C and T; 20 had M and C; 30 had C and T; 25 had M and T; 12 had M only; 5 had C only; and 8 had T only. Using a Venn diagram find how many did not take any of the three drinks. **[1978]**

Answer Key

Topic-1 : Sets, Types of Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1. (d) 2. (d)

Topic-2 : Venn Diagrams, De Morgan's Law, Practical Problem

1. (c) 2. (c)

Hints & Solutions

Topic-1: Sets, Types of Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1. **(d)** $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\Rightarrow \sin\theta = (\sqrt{2}+1)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1$

Now, $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\Rightarrow \cos\theta = (\sqrt{2}-1)\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\Rightarrow \tan\theta = \sqrt{2}+1$

$\therefore$ P = Q

2. **(d)** $S = \{1, 2, 3, 4\}$

Let A and B be disjoint subsets of S

Now for any element $a \in S$, has got three possibilities either, it is in A or B or none

$\Rightarrow$ For every element out of 4 elements there are three choices

$\therefore$ Total options $= 3^4 = 81$

Here $A \neq B$ except when $A = B = \phi$

$\therefore 81 - 1 = 80$ ordered pairs (A, B) are there for which $A \neq B$

Hence total number of unordered pairs of disjoint

subsets $= \frac{80}{2} + 1 = 41$

Topic-2: Venn Diagrams, De Morgan's Law, Practical Problem

1. **(c)** $X \cap (X \cup Y)^c = X \cap (X^c \cap Y^c) = (X \cap X^c) \cap Y^c$

$= \phi \cap Y^c = \phi \quad (\because X \cap X^c = \phi)$

2. **(c)** Let the number of newspapers which are read be n. Then $60\,n = (300)(5) \Rightarrow n = 25$

3. Given that $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ ....(i)

and each $A_i'^s$ contain 5 elements

So, total number of elements in $A_i = 5 \times 30 = 150$.

Since each element of S belongs to exactly ten of the A_i's.

$\therefore\ n(S) = n\left[\bigcup_{i=1}^{30} A_i\right] = \frac{150}{10} = 15$ ...(ii)

Now, each B_j^s contain 3 elements

So, total number of elements in $B_j = 3 \times n = 3n$.

Since each element of S belongs to exactly nine of the B_j's.

$\therefore\ n(S) = n\left[\bigcup_{j=1}^{n} B_j\right] = \frac{3n}{9}$ ...(iii)

from (ii) and (iii)

$\frac{3n}{9} = 15 \Rightarrow n = 45.$

4. (i) $n(A) = 3, n(B) = 6$

We know that, $n(A \cup B) \geq \max.(n(A), n(B))$

$\Rightarrow n(A \cup B) \geq 6$

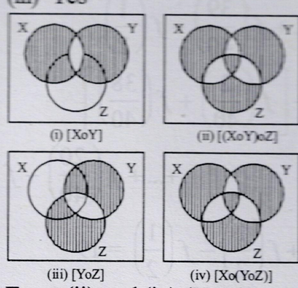
$\therefore$ Minimum number of elements in the set $A \cup B$ is 6.

(ii) $R \times (P^c \cup Q^c)^c = R \times (P \cap Q)$

$= (R \times P) \cap (R \times Q)\ [\because (A \cup B)^c = A^c \cap B^c]$

$\therefore$ Given equality is true.

(iii) Yes



From (ii) and (iv) (XoY)oZ = Xo(YoZ)

5. We have

$n(U) = 100$, where U stands for universal set

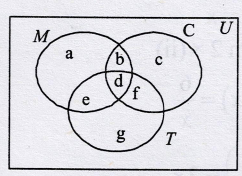
$n(M \cap C \cap T) = d = 10$; $n(M \cap C) = b + d = 20$;.

$n(C \cap T) = d + f = 30$; $n(M \cap T) = d + e = 25$;

$\Rightarrow b = 10, f = 20$ and $e = 15$

n(only M) $= a = 12$; n(only C) $= c = 5$; n(only T) $= g = 8$

Filling all the entries we obtain the Venn diagram as shown :



$\therefore\ n(M \cap C \cup T) = a + b + c + d + e + f + g$

$= 12 + 10 + 5 + 15 + 10 + 20 + 8 = 80$

$\therefore\ n(M \cup C \cup T)' = 100 - 80 = 20$